HW #3 ; Sec 4.1, Problems 20 - 23 SOLUTIONS with the replaced instructions:

Sec 4.1, # 20.

TO PROVE: For all integers m, if $m > 1$, then $0 < \frac{1}{m} < 1$.

Proof: Let m be any integer.
Suppose $m > 1$.
[NTS: $0 < \frac{1}{m} < 1$]

...

THEREFORE, $0 < \frac{1}{m} < 1$.
$\therefore$ For all integers m, if $m > 1$, then $0 < \frac{1}{m} < 1$, by Direct Proof.
QED.

#21. To Prove: For all real numbers x, if $x > 1$, then $x^2 > x$.

Proof: Let x be any real number.
Suppose $x > 1$.
[NTS: $x^2 > x$]

...

$\therefore$ $x^2 > x$
Therefore, for all real numbers x, if $x > 1$, then $x^2 > x$, by Direct Proof.
QED.

Sec 4.1, # 22.

<u>To prove</u>: For all integers m and n,
if $mn = 1$, then $m = n = 1$ or $m = n = -1$.

Proof: Let m and n be any integers.
Suppose $mn = 1$.
[NTS: $m = n = 1$ or $m = n = -1$]

...

$\therefore$ $m = n = 1$ or $m = n = -1$.

$\therefore$ For all integers m and n,
if $mn = 1$, then $m = n = 1$ or $m = n = -1$,
by Direct Proof.

QED

23.

To Prove: For all real numbers x,
if $0 < x < 1$, then $x^2 < x$.

Proof: Let x be any real number.
Suppose $0 < x < 1$.
[NTS: $x^2 < x$]

...

$\therefore$ $x^2 < x$.

Therefore, for all real numbers x,
if $0 < x < 1$, then $x^2 < x$, by Direct Proof.

QED.